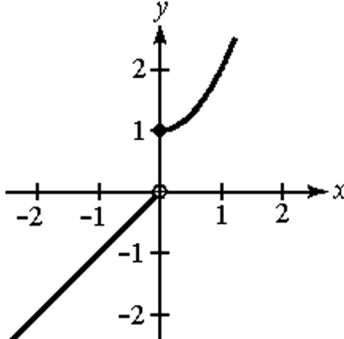
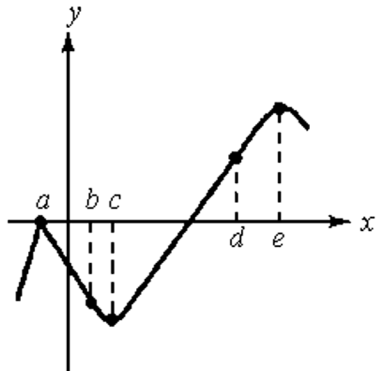
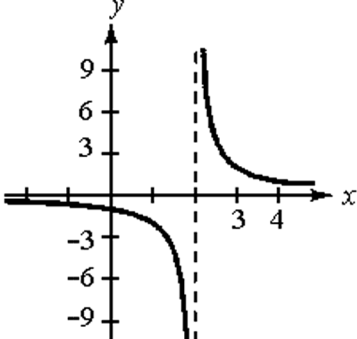
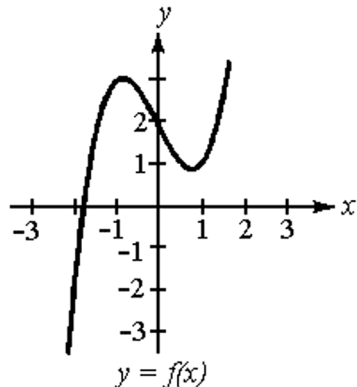


Calculus 1st Semester Final Review

<p>1. Use the graph to find <math>\lim_{x \rightarrow 0} f(x)</math> (if it exists).</p> 	<p>9. Determine the value of <math>c</math> so that <math>f(x)</math> is continuous on the entire real line if <math>f(x) = \begin{cases} x^2, &amp; x \leq 3 \\ c/x, &amp; x &gt; 3 \end{cases}</math>.</p>
<p>2. Find the limit: <math>\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 3}</math>.</p>	<p>10. Find the limit: <math>\lim_{x \rightarrow 3} \frac{3}{x^2 - 6x + 9}</math>.</p>
<p>3. Find the limit: <math>\lim_{x \rightarrow 1} f(x)</math> if <math>f(x) = \begin{cases} x^2 + 4, &amp; x \neq 1 \\ 2, &amp; x = 1 \end{cases}</math>.</p>	<p>11. Find all vertical asymptotes of the graph of <math>f(x) = \frac{x^2 + 3x - 1}{x + 7}</math>.</p>
<p>4. Find the limit: <math>\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}</math>.</p>	<p>12. Find the vertical asymptote(s) of <math>f(x) = \frac{x^2 - x - 2}{x^2 + x - 6}</math>.</p>
<p>5. Find the limit: <math>\lim_{x \rightarrow 1} \frac{1 - \sqrt{2x^2 - 1}}{x - 1}</math>.</p>	<p>13. At each point indicated on the graph, determine whether the value of the derivative is positive, negative, zero, or if the function has no derivative.</p> 
<p>6. Let <math>f(x) = \begin{cases} x^2 + 1, &amp; x \leq 0 \\ 2x - 3, &amp; x &gt; 0 \end{cases}</math>. Find each limit (if it exists).</p> <p>a. <math>\lim_{x \rightarrow 0^-} f(x)</math></p> <p>b. <math>\lim_{x \rightarrow 0^+} f(x)</math></p> <p>c. <math>\lim_{x \rightarrow 0} f(x)</math></p>	<p>14. Use the definition of a derivative to calculate the derivative of <math>f(x) = x^2 + 2</math>.</p>
<p>7. Find the values of <math>x</math> for which <math>f(x) = \frac{x - 2}{x^2 - 4}</math> is discontinuous and label these discontinuities as removable or nonremovable.</p>	<p>15. Find an equation of the tangent line to the graph of <math>f(x) = 3x^3 + 2x</math> when <math>x = 1</math>.</p>
<p>8. Let <math>f(x) = \frac{5}{x - 1}</math> and <math>g(x) = x^4</math>.</p> <p>a. Find <math>f(g(x))</math>.</p> <p>b. Find all values of <math>x</math> for which <math>f(g(x))</math> is discontinuous.</p>	<p>16. Find the values of <math>x</math> for all points on the graph of <math>f(x) = x^3 - 2x^2 + 5x - 16</math> at which the slope of the tangent line is 4.</p>
	<p>17. Find all points at which the graph of <math>f(x) = x^3 - 3x</math> has horizontal tangent lines.</p>
	<p>18. The position function for an object is given by <math>s(t) = 6t^2 + 240t</math>, where <math>s</math> is measured in feet and <math>t</math> is measured in seconds. Find the velocity of the object when <math>t = 2</math> seconds.</p>
	<p>19. Differentiate: <math>y = \frac{2x}{(1 - 3x^2)^2}</math>.</p>
	<p>20. Calculate <math>\frac{d^2 y}{dx^2}</math> if <math>y = \frac{1 - x}{2 - x}</math>.</p>

21. Find the derivative of $y = \sqrt[3]{x^2 + x}$ .	36. Determine whether the Mean Value Theorem applies to $f(x) = 3x - x^2$ on the interval $[2, 3]$ . If the Mean Value Theorem can be applied, find all value(s) of $c$ in the interval such that $f'(c) = \frac{f(3) - f(2)}{3 - 2}$ . If the Mean Value Theorem does not apply, state why.
22. Find the derivative of $y = (x^2 + 2x + 5)^6$ .	37. Find the open intervals on which $f(x) = \frac{1}{x^2}$ is increasing or decreasing.
23. Find $\frac{dy}{dx}$ for $y = \sqrt{2x+1}(x^3)$ .	38. Find the open intervals on which $f(x) = x^3 - 3x^2$ is increasing or decreasing.
24. The position equation for the movement of a particle is given by $s = (t^3 + 1)^2$ where $s$ is measured in feet and $t$ is measured in seconds. Find the acceleration of this particle at 1 second.	39. Use the graph to identify the open intervals on which the function is increasing or decreasing.
25. Find $\frac{dy}{dx}$ if $y = \frac{x}{x+y}$ .	
26. Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2 + xy + y^2 = 5$ .	40. Find all relative extrema of $y = \frac{2x}{(x+4)^3}$ .
27. Find the slope of the curve $y^4 - xy^2 = x$ at the point $(\frac{1}{2}, 1)$ .	41. Find the relative minimum and relative maximum for $f(x) = 2x^3 + 3x^2 - 12x$ .
28. The radius of a circle is increasing at the rate of 5 inches per minute. At what rate is the area increasing when the radius is 10 inches?	42. Use the first derivative test to find the $x$ -values that give relative extrema for $f(x) = -x^4 + 2x^3$ .
29. Air is being pumped into a spherical balloon at a rate of 28 cubic feet per minute. At what rate is the radius changing when the radius is 3 feet? $(V = \frac{4}{3}\pi r^3)$	43. Let $f(x) = \frac{x}{1-x}$ . Show that $f$ has no critical numbers.
30. A point moves along the curve $y = \frac{-x+4x-3}{10}$ so that the $y$ value is decreasing at a rate of 3 units per second. Find the instantaneous rate of change of $x$ with respect to time at the point on the curve where $x = 5$ .	44. A differentiable function $f$ has only one critical number: $x = -3$ . Identify the relative extrema of $f$ at $(-3, f(-3))$ if $f'(-4) = \frac{1}{2}$ and $f'(-2) = -1$ .
31. Find all critical numbers for the function: $f(x) = x\sqrt{2x+1}$ .	45. Find the intervals on which the graph of the function $f(x) = x^4 - 4x^3 + 2$ is concave upward or downward. Then find all points of inflection for the function.
32. Find all critical numbers for the function: $f(x) = 3x^4 - 4x^3$ .	46. Find all points of inflection of the graph of the function $f(x) = 2x(x-4)^3$ .
33. Find the minimum and maximum values of $f(x) = x^2 - 2x + 1$ on the interval $[0, 3]$ .	47. Find all points of inflection of the graph of the function $f(x) = x^3 - 3x^2 - x + 7$ .
34. Consider $f(x) = \frac{1}{(x-3)^2}$ . a. Sketch the graph of $f(x)$ . b. Calculate $f(2)$ and $f(4)$ . c. State why Rolle's Theorem does not apply to $f$ on the interval $[2, 4]$ .	48. Let $f(x) = x^3 - x^2 + 3$ . Use the Second Derivative Test to determine which critical numbers, if any, give relative extrema.
35. Decide whether Rolle's Theorem can be applied to $f(x) = x^4 - 4x^3 + 4x^2 + 1$ on the interval $[-1, 3]$ . If Rolle's Theorem can be applied, find all value(s), $c$ , in the interval such that $f'(c) = 0$ . If Rolle's Theorem cannot be applied, state why.	

49. The graph of a polynomial function,  $f$ , is given. On the same coordinate axes sketch  $f'$  and  $f''$ .



50. Find the horizontal asymptote for  $f(x) = \frac{2x^2 + x - 7}{x^2 - 1}$ .

51. Find the limit:  $\lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{4x^2 - 1}}$ .

52. An open box is to be made from a square piece of material, 12 inches on each side, by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made in this manner.

53. Use the techniques learned in this chapter to sketch the graph of  $f(x) = x^3 + x^2 - 6x$ .

54. A rancher has 300 feet of fencing to enclose a pasture bordered on one side by a river. The river side of the pasture needs no fence. Find the dimensions of the pasture that will produce a pasture with a maximum area.

55. A manufacturer determines that  $x$  employees on a certain production line will produce  $y$  units per month where  $y = 75x^2 - 0.2x^4$ . To obtain maximum monthly production, how many employees should be assigned to the production line?

56. The volume of a cube is claimed to be 27 cubic inches, correct to within  $0.027 \text{ in.}^3$ . Use differentials to estimate the propagated error in the measurement of the side of the cube.

Calculus 1st Semester Final Review

Reference: [6.6]

[1] The limit does not exist.

Reference: [7.8]

[2] 0

Reference: [7.14]

[3] 5

Reference: [7.39]

[4] 3

Reference: [7.46]

[5] -2

Reference: [8.10]

a. 1

b. -3

[6] c. The limit does not exist.

Reference: [8.20]

[7]  $x = 2$ , removable;  $x = -2$ , nonremovable

Reference: [8.23]

a.  $\frac{5}{x^4 - 1}$

[8] b. -1, 1

Reference: [8.26]

[9] 27

Reference: [9.8]

[10]  $\infty$

Reference: [9.23]

[11]  $x = -7$

Reference: [9.25]

[12]  $x = -3$

Reference: [10.5]

a. no derivative

b. negative

c. zero

d. positive

[13] e. zero

Reference: [10.11]

[14]  $-2x$

Reference: [11.23]

[15]  $y = 11x - 6$

Reference: [11.26]

[16]  $\frac{1}{3}, 1$

Reference: [11.28]

[17]  $(1, -2), (-1, 2)$

Reference: [11.40]

[18] 264 ft/sec

Reference: [12.2]

[19]  $\frac{6x^2 + 2}{(1 - 3x^2)^2}$

Reference: [12.25]

[20]  $\frac{-2}{(2 - x)^3}$

Reference: [13.2]

[21]  $\frac{2x + 1}{3(x^2 + x)^{2/3}}$

Reference: [13.3]

[22]  $12(x + 1)(x^2 + 2x + 5)^5$

Reference: [13.11]

[23]  $\frac{x^2(7x + 3)}{\sqrt{2x + 1}}$

Reference: [13.34]

[24] 42 ft/sec<sup>2</sup>

Reference: [14.3]

[25]  $\frac{y}{(x + y)^2 + x}$

Reference: [14.10]

[26]  $\frac{-2x - y}{x + 2y}$

Reference: [14.29]

$$\frac{2}{3}$$

[27] 3

Reference: [15.5]

[28]  $100\pi \text{ in.}^2/\text{min}$

Reference: [15.14]

$$\frac{7}{9\pi} \text{ ft/min}$$

[29]  $9\pi$

Reference: [15.19]

[30] 5 units/sec

Reference: [16.3]

$$-\frac{1}{3}, -\frac{1}{2}$$

[31]

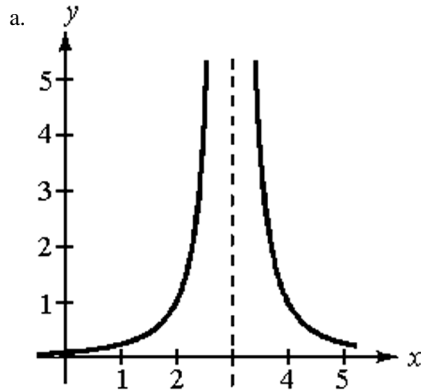
Reference: [16.4]

[32]  $x = 0, 1$

Reference: [16.17]

[33] Minimum at (1, 0); Maximum at (3, 4)

Reference: [17.3]



b.  $f(2) = f(4) = 1$

[34]  $f$  is not continuous on  $[2, 4]$

Reference: [17.7]

[35] Rolle's Theorem applies;  $c = 0, 1,$  and  $2.$

Reference: [17.14]

The Mean Value Theorem applies;  $c = \frac{5}{2}$ .

[36]

Reference: [18.3]

[37] Increasing  $(-\infty, 0)$ ; decreasing  $(0, \infty)$

Reference: [18.4]

[38] Increasing  $(-\infty, 0)$  and  $(2, \infty)$ ; decreasing  $(0, 2)$

Reference: [18.10]

[39] Decreasing  $(-\infty, 2)$  and  $(2, \infty)$

Reference: [18.16]

[40]  $\left(2, \frac{1}{54}\right)$ , relative maximum

Reference: [18.17]

[41] Relative maximum:  $(-2, 20)$ ; relative minimum:  $(1, -7)$

Reference: [18.22]

[42] Relative maximum at  $x = \frac{3}{2}$

Reference: [18.27]

$$f'(x) = \frac{1}{(1-x)^2} \neq 0 \text{ for all } x \neq 1.$$

[43]  $f'(x)$  is undefined at  $x = 1$ , a vertical asymptote.

Reference: [18.28]

[44] Relative maximum

Reference: [19.4]

Concave upward:  $(-\infty, 0), (2, \infty)$   
Concave downward:  $(0, 2)$

[45] Points of inflection:  $(0, 2)$  and  $(2, -14)$

Reference: [19.14]

[46]  $(4, 0), (2, -32)$

Reference: [19.18]

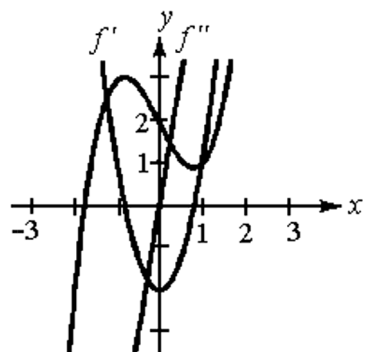
[47]  $(1, 4)$

Reference: [19.21]

[48]  $x = 0$ , relative maximum;  $x = \frac{2}{3}$ , relative minimum

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Reference: [19.32]



[49] \_\_\_\_\_

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Reference: [20.3]

[50]  $y = 2$  \_\_\_\_\_

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Reference: [20.17]

[51]  $\infty$  \_\_\_\_\_

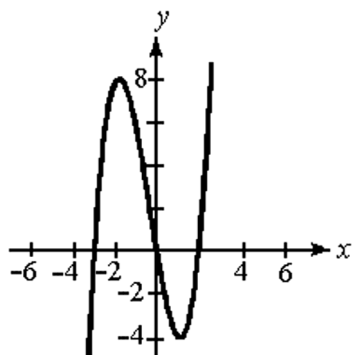
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Reference: [22.17]

[52]  $8^2(2) = 128$  cubic inches \_\_\_\_\_

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Reference: [21.18]



[53] \_\_\_\_\_

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Reference: [22.8]

[54] 75 feet by 150 feet \_\_\_\_\_

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Reference: [22.28]

[55] 14 \_\_\_\_\_

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Reference: [24.3]

[56]  $\pm 0.001$  in. \_\_\_\_\_